JOURNAL OF AIRCRAFT
Vol. 31, No. 2, March–April 1994

Aircraft Fleet Maintenance Based on Structural Reliability Analysis

J. N. Yang*
University of California, Irvine, California 92717
and
S. D. Manning†
Lockheed/Fort Worth Company, Fort Worth, Texas 76101

A stochastic crack growth analysis methodology, based on the lognormal random variable model, is described and demonstrated for two practical aircraft structural maintenance applications. In the first application, a reliability centered maintenance analysis for evaluating aircraft structural maintenance and supportability requirements and options in terms of risk or reliability is demonstrated. The sensitivity of initial and reinspection intervals to variations in analysis variables is investigated using a cutout in an aluminum-lithium cheek frame. The second application is the maintenance scheduling for a fleet of aircraft on a calendar year basis in terms of risk or reliability. The crack growth life dispersion due to material, service usage severity, and aircraft utilization rate on the fleet maintenance schedule are accounted for. Aircraft fleet tracking data are used and the sensitivity of fleet maintenance requirements to variations in the design stress levels and other variables are investigated and presented.

Nomenclature

a_D = inspection limit, i.e., the crack size that can be detected by a particular NDE system with 90% probability and 95% confidence
$a(t)$ = crack size at time t
$\bar{a}_0$ = deterministic initial flaw size
$F_{T(x_1)}(\tau)$ = distribution function of time to reach a given crack size x_1
g = deterministic crack growth rate function
P_a = conditional probability of a class A mishap (catastrophic failure) under the condition that a control point fails
$P_f(\tau)$ = probability of failure of a control point in $(0, \tau)$
$p(i, \tau)$ = probability of crack exceedance, i.e., the probability that $a(\tau)$ will exceed x_1
R_a = risk rate
$\bar{R}$ = median utilization rate of a fleet of aircraft in flight hours per day
S = stress level
$t_n(I)$ = total number of flight hours accumulated by the Ith aircraft series number
$\bar{t}(I, x_2)$ = median service time to reach crack size x_2 for the Ith aircraft series number
$\bar{t}(x_1)$ = median service time to reach given x_1 from $\bar{a}_0$
X = lognormal random variable
x_1, x_2 = any given crack size
γ = risk level
σ_z = crack growth life dispersion parameter
τ = any given service time
τ_r = reinspection interval
τ_1 = initial inspection interval

Introduction

RISK assessment methods for in-service aircraft have received considerable attention.[1–8] Aircraft structural inspection and repair maintenance is essential for ensuring the structural integrity against the consequences of fatigue cracking. Probabilistic methods are useful for evaluating aircraft structural maintenance requirements and options in terms of risk. In-service inspections of airframes have indicated that fatigue cracks commonly occur in structural details, such as fastener holes, cutouts, fillets, etc., and aircraft structures contain thousands of such details. Field inspections and laboratory fatigue test results show variations in the crack growth rate for individual aircraft. Such variations should be accounted for in the fleet maintenance plan so that timely maintenance and repair can be appropriately scheduled in terms of risk or reliability.

Recently, stochastic crack growth approaches have received considerable attention, and various stochastic crack growth models have been proposed in the literature.[10–24] In this article, the lognormal random variable model, previously developed by Yang et al.,[9,11,14,24–26] for predicting the crack growth accumulation in metallic structures, will be described and applied to aircraft maintenance. This simple model has been used for the stochastic crack growth analysis of fastener holes where the initial flaw size is a single value,[9,11,14] and where the initial flaw size is a random variable.[6–8,25,26] It has also been extended to a multisegment approach which allows the crack growth life dispersion to vary for selected crack size ranges.[24]

The lognormal random variable model results in conservative crack growth damage predictions because its statistical dispersion is the largest among the class of general random processes. Nevertheless, the lognormal random variable model is very attractive for scheduling airframe maintenance in terms of risk because 1) it is mathematically very simple and easy to implement; 2) it is of a conservative nature; 3) it accounts for the effects of variations in material crack growth resistance, usage severity, utilization rate, etc., on crack growth accumulations; 4) it can be implemented using a deterministic general crack growth computer program; 5) a small number of replicate fatigue test results are adequate to calibrate the model since it does not require a correlation distance parameter; and 6) reasonable crack growth predictions for structural details have been previously obtained for both coupon specimens and full-scale aircraft structures.[11,25,26]

Received July 24, 1992; revision received Jan. 13, 1993; accepted for publication Jan. 25, 1993.

*Professor, Department of Civil Engineering. Member AIAA.
†Structures Technology Staff, Structures Technology Division.

The objective of this article is to demonstrate two practical applications of the lognormal stochastic crack growth model developed and extended by Yang et al.[9,11,14,24-26] to 1) the reliability centered maintenance analysis method for determining structural inspection requirements (i.e., initial and reinspection intervals) for metallic airframes; and 2) scheduling fleet maintenance in terms of calendar time and risk. In the first application, the sensitivity of inspection intervals to variations in stress level, risk rate, probability of a class A mishap, etc., are evaluated. In the second application, the crack growth variability due to material crack resistance, usage severity, aircraft utilization rate, etc., are accounted for. The individual aircraft tracking (IAT) results for a fighter aircraft are effectively used to develop fleet maintenance schedules in terms of risk or reliability. Useful formats for evaluating fleet maintenance options and tradeoffs are presented.

Formulation

Stochastic Crack Growth Model

Various stochastic crack growth models have been proposed and investigated in the literature,[10-24] mainly for metallic materials and superalloys. In particular, a book edited by Provan[10] contains extensive literature on such models. Early works on statistical crack growth are described in Refs. 12 and 13. Bogdanoff and Kozin[20,21] proposed a model that the evolution of $a(t)$ is a discrete Markov chain. Most of the stochastic models investigated are based on the randomization of a well-known deterministic crack growth rate function $g(\Delta K, R, a)$, i.e.

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = X(\eta)g(\Delta K, R, a) \tag{1}$$

in which t denotes either flights or flight hours, ΔK is the stress intensity range, R is the stress ratio, and $X(\eta)$ is a random process. In Eq. (1), $g(\Delta K, R, a)$ can be any deterministic crack growth rate function.[27] Yang et al.[9,16] suggested that $X(\eta)$ can be a random process of time t, i.e., $X(\eta) = X(t)$, or a random process of ΔK,[14] i.e., $X(\eta) = X(\Delta K)$. Of course, ΔK is a function of t. The random process model $X(t)$ has been investigated extensively considering $X(t)$ a Poisson process,[16,18] a diffusion Markov process,[17,23] a lognormal random process,[11] and a lognormal random variable,[9,11,14] etc. The random process model $X(\Delta K)$ has been investigated.[22] Recently, $X(\eta)$ has been considered as a random process of $a(t)$, i.e., $X(\eta) = X(a)$.[4,24]

When $X(\eta)$ is a stationary random process of either t, ΔK, or $a(t)$, an autocorrelation function or the correlation distance parameter should be determined from test data, in addition to the mean value and the standard deviation. Such information is difficult to obtain due to the limited experimental test results available.

The lognormal random variable model is a special case of the lognormal random process model where the correlation distance is infinite. Extensive crack growth test data for fastener holes indicate that the correlation distance is very long.[11,16-18,24] Hence, the lognormal random variable model is quite reasonable.

The lognormal random variable model can be written as

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = Xg(\Delta K, R, a) \tag{2}$$

in which X is a median value of 1.0 and a log standard deviation σ_z for $Z = \log X$. Thus, the deterministic crack growth rate equation

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = g(\Delta K, R, a) \tag{3}$$

represents the median crack growth rate; whereas X accounts for the statistical variability of the crack growth accumulation. The distribution function for X is given by

$$F_X(x) = P(X \le x) = \Phi(\ln x/\sigma_z) \tag{4}$$

in which $\Phi(\ )$ is the standardized normal distribution function.

Equation (3) has been used extensively by industry for damage tolerance and durability analyses. Integrating Eq. (3) from $\bar{a}_0$ to x_1, one obtains

$$\int_0^{\bar{t}(x_1)} \mathrm{d}t = \bar{t}(x_1) = \int_{\bar{a}_0}^{x_1} \frac{\mathrm{d}a}{g(\Delta K, R, a)} \tag{5}$$

In Eq. (5), $\bar{t}(x_1)$ is referred to as the "median crack growth curve." The right side of Eq. (5) can be obtained using any available general crack growth computer code. Either a cycle-by-cycle or flight-by-flight numerical integration procedure can be used. Various $g(\Delta K, R, a)$ have been proposed in the literature,[27] such as Forman equation, Walker equation, hyberbolic sine equation, etc. Any type of the crack growth rate function and any retardation model can be used in Eq. (5).

Let $T(x_1)$ be a random variable denoting the service time to reach x_1 from $\bar{a}_0$. Integrating Eq. (2) from $\bar{a}_0$ to x_1, one obtains

$$X\int_0^{T(x_1)} \mathrm{d}t = \int_{a(0)}^{x_1} \frac{\mathrm{d}a}{g(\Delta K, R, a)} \tag{6}$$

Substituting Eq. (5) into Eq. (6), one obtains

$$T(x_1) = \bar{t}(x_1)/X \tag{7}$$

The cumulative distribution function of the service time $T(x_1)$ to reach x_1 from $\bar{a}_0$, denoted by $F_{T(x_1)}(\tau) = P[T(x_1) \le \tau]$, is derived from Eq. (4) through the transformation of Eq. (7) as follows:

$$F_{T(x_1)}(\tau) = \Phi\left[\frac{\ln \tau - \ln \bar{t}(x_1)}{\sigma_z}\right] \tag{8}$$

The probability that the crack size $a(\tau)$ at a structural detail (e.g., a fastener hole), in the ith stress region, will exceed x_1 in the service time $(0, \tau)$ is referred to as the "probability of crack exceedance," denoted by $p(i, \tau)$, i.e., $p(i, \tau) = P[a(\tau) > x_1]$. $p(i, \tau)$ is identical to the probability that the service time $T(x_1)$ to reach x_1 will be smaller than τ, which is the distribution function of $T(x_1)$ evaluated at τ, i.e.

$$p(i, \tau) = P[a(\tau) > x_1] = P[T(x_1) \le \tau] = F_{T(x_1)}(\tau) = \Phi\left[\frac{\ln \tau - \ln \bar{t}(x_1)}{\sigma_z}\right] \tag{9}$$

Figure 1 clearly demonstrates the identical relation between $p(i, \tau)$ and $F_{T(x_1)}(\tau)$.

Reliability Centered Maintenance Analysis

The risk rate [or probability of a class A mishap per flight hour (FH)] of an aircraft is expressed as follows:

$$R_a = P_a P_f(\tau)/\tau \tag{10}$$

In Eq. (10), R_a is the risk rate of an aircraft in $(0, \tau)$, where 0 denotes the time of last preventive maintenance or initial production; and τ is the service time since last preventive maintenance or initial production.

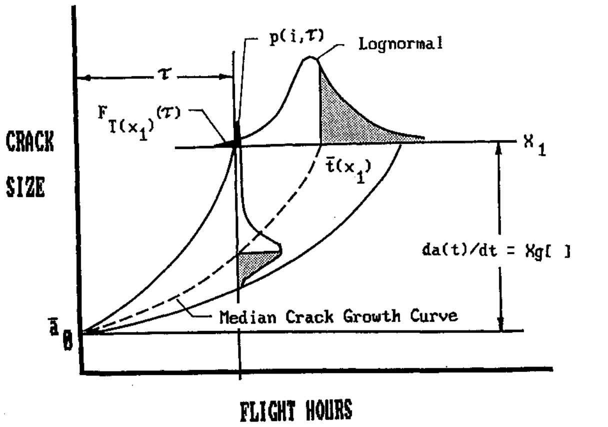

Fig. 1 Stochastic crack growth analysis approach.

$P_f(\tau)$ of a control point in $(0, \tau)$ is the probability that $a(\tau)$ at τ will exceed the critical $x_1 = a_{cr}$, i.e., $P_f(\tau) = P[a(\tau) > x_1] = p(i, \tau) = F_{T(x_1)}(\tau)$. Hence, Eq. (10) can be written as

$$F_{T(x_1)}(\tau)/\tau = R_a/P_a \tag{11}$$

in which $x_1 = a_{cr}$ and $F_{T(x_1)}(\tau)$ is given by Eq. (8).

P_a is estimated using an event tree logic to predict the consequence probability should a control point fail.[28] Similarly, the allowable R_a is based on the applicable air vehicle specification, historical data for structural maintenance, and judgment. The probability of failure of a control point, $F_{T(x_1)}(\tau)$, is obtained from Eq. (8). Thus, Eq. (11) will be used to determine the initial and reinspection intervals and to evaluate inspection technique options for selected control points. Substituting Eq. (8) into Eq. (11), one obtains a nonlinear equation for τ:

$$\Phi\{[\ln \tau - \ln \bar{t}(x_1)]/\sigma_z\}/\tau = R_a/P_a \tag{12}$$

Theoretically, the initial inspection interval, $\tau = \tau_1$, for the control point can be determined using Eq. (12) and the following input information: 1) the applicable R_a, e.g., $R_{a(\max)}$ or $R_{a(\min)}$; 2) the conditional probability of P_a; 3) $\bar{a}_0$; 4) the critical flaw size $a_{cr} = x_1$; 5) the median crack growth curve $\bar{t}(x_1)$; and 6) σ_z. However, since Eq. (12) is a nonlinear equation for $\tau = \tau_1$, it is more convenient to compute the initial inspection interval using Eqs. (8) and (11) and the following iterative procedures: 1) given R_a and P_a, the probability of failure of the control point per flight hour $F_{T(x_1)}(\tau)/\tau$ is computed from Eq. (11); and 2) given $\bar{a}_0$, $a_{cr} = x_1$, $\bar{t}(x_1)$, and σ_z, $F_{T(x_1)}(\tau)$ can be computed from Eq. (8) for τ. By trial and error, the required service time $\tau = \tau_1$ can be determined which satisfies Eq. (11).

τ_r can be determined using the same iterative procedure described above by replacing $\bar{a}_0$ with a_D. a_D is the crack size associated with 90% detection probability and 95% confidence for the particular NDE technique used. Another procedure for determining τ_1 and τ_r, based on probabilistic crack growth trajectories, can also be used.[28]

As mentioned previously, $\bar{t}(x_1)$ can be obtained using the deterministic crack growth analysis, Eq. (5), and a suitable analytical crack growth computer code.[29] The log standard deviation σ_z in Eq. (8) represents the crack growth life dispersion. It can be estimated using either a set of sample crack growth trajectories,[26] or the method described in the next section.

Fleet Maintenance Schedule on a Calendar Year Basis

In Eq. (2), X accounts for the overall statistical variability of the crack growth rate. The log standard deviation σ_z of $Z = \log X$ represents the overall crack growth life dispersion from various sources. For a fleet of aircraft, these sources of variabilities include the material crack growth resistance, usage severity (load spectra variability), aircraft utilization rate, environmental effects, etc. As a result, the variability from different sources should be determined separately using available service data, such as the IAT results, as follows.

Yang et al.[6,7] proposed that X be modeled by

$$X = H_1 H_2 H_3 \tag{13}$$

in which H_1, H_2, and H_3 are all statistically independent lognormal random variables with a median value of 1.0. H_1, H_2, and H_3 account for the crack growth life variabilities due to material, usage severity, and aircraft utilization rate, respectively. Then, it follows from Eq. (13) that the overall crack growth life variability σ_z, appearing in Eqs. (4), (8), and (9), is given by

$$\sigma_z = [\sigma_{z1}^2 + \sigma_{z2}^2 + \sigma_{z3}^2]^{1/2} \tag{14}$$

in which σ_{z1}, σ_{z2}, and σ_{z3} are the standard deviations, respectively, of the normal random variables $Z_1 = \log H_1$, $Z_2 = \log H_2$, and $Z_3 = \log H_3$. σ_{z1}, σ_{z2}, and σ_{z3} are referred to as the log standard deviations of H_1, H_2, and H_3, respectively.

Note that H_1, H_2, and H_3 are positive random variables. Individual aircraft tracking results indicate that these random variables follow the lognormal distribution reasonably well. Furthermore, the statistical model for X given by Eq. (13) is quite reasonable. σ_{z1}, due to material crack resistance, can be obtained from laboratory experimental data using the maximum likelihood method.[24–26] Methods for determining σ_{z2} and σ_{z3} using IAT results are described in the following.

Let $\bar{t}(I, x_2)$ be the median number of flight hours at a given IAT control point. $\bar{t}(I, x_2)$ is computed using Eq. (5) and the usage severity parameters obtained for the Ith aircraft from the individual aircraft tracking results as shown in Fig. 2a. Due to usage severity (load spectra) variability, $\bar{t}(I, x_2)$ varies from Ith aircraft to Jth aircraft. If H_1 and H_3 have a median value of 1.0, it follows from Eq. (8) with $\sigma_z = \sigma_{z2}$ that σ_{z2} due to usage severity can be estimated using the maximum likelihood method[24–26] as follows: $\sigma_{z2} = [YY - Y^2]^{1/2}$ in which $Y = \Sigma\,[\ln \bar{t}(I, x_2)/N]$ and $YY = \Sigma\,[\ln \bar{t}(I, x_2)]^2/N$, where N is the total number of aircraft serial numbers in the fleet considered, and the summation is from 1 to N.

Let $t_n(I)$ be the total number of FH accumulated on the recording date $D_n(I)$, and $D_d(I)$ be the delivery date for the

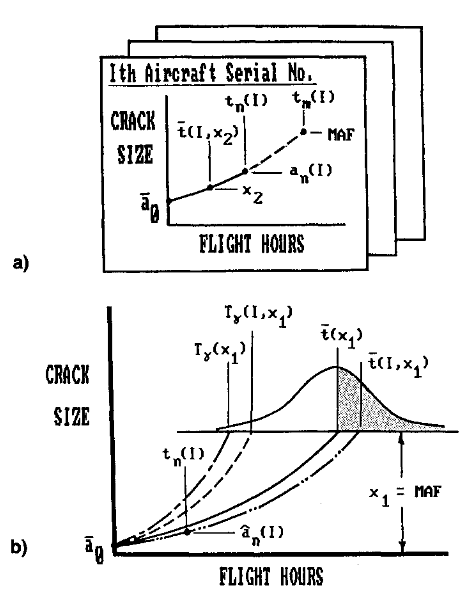

Fig. 2 Crack growth analysis results for IAT control point and structural detail: a) IAT results for IAT control point and b) crack growth analysis for structural detail.

Ith aircraft serial number. Then, the utilization rate (flight hours per day) of the Ith aircraft, denoted by $R(I)$, is given by

$$R(I) = t_n(I)/[D_n(I) - D_d(I)] \tag{15}$$

in which $t_n(I)$ is available from the individual aircraft tracking results (Fig. 2a).

The lognormal random variable H_3 represents the normalized utilization rate with a median value of 1.0. Therefore, the normalized $R(I)$, denoted by $R^*(I)$, is a sample value of H_3

$$\ell n\, R^*(I) = \ell n\, R(I) - \frac{1}{N}\sum_{I=1}^{N} \ell n\, R(I) \tag{16}$$

The standard deviation, σ_{z3}, of $Z_3 = \log H_3$ is obtained from the maximum likelihood estimate as follows: $\sigma_{z2} = [ZZ - Z^2]$,$^{1/2}$ in which $Z = \Sigma\, \ell n\, R^*(I)/N$ and $ZZ = \Sigma\, [\ell n\, R^*(I)^2/N$.

Since the normalized utilization rate, H_3, is a lognormal random variable, R for a fleet of aircraft is also a lognormal random variable with a median value $\bar{R}$, given by

$$\bar{R} = \exp\left[\frac{1}{N}\sum_{I=1}^{N} \ell n\, R(I)\right] \tag{17}$$

in which $R(I)$ is obtained from Eq. (15).

The results used to determine σ_{z2} and σ_{z3} described above are for a given IAT control point. However, the particular control point considered for the maintenance schedule, in general, is not the same as the IAT control point. Hence, there will be a different stress level at the control point of interest and at the IAT control point. In establishing the maintenance schedule for the control point of interest, two approaches are proposed in the following.

In the first approach, the only individual aircraft tracking result available for the particular control point of interest in each aircraft is $t_n(I)$, i.e., the total number of flight hours accumulated by the Ith aircraft serial number on the recording date $D_n(I)$. This is usually the case in practice. Based on the fleet average load spectrum for the entire fleet of aircraft, the time $\bar{t}(x_1)$ to reach the maintenance action flaw (MAF) size, x_1, from $\bar{a}_0$ can be computed from Eq. (5) using a general crack growth computer code.[29] $\bar{t}(x_1)$ is schematically shown in Fig. 2b. Let $T_\gamma(x_1)$ be the γ percentile of the service time, $T(x_1)$, to reach the maintenance action flaw size x_1, as shown schematically by a dashed-dotted curve in Fig. 2b. In other words, the probability is $\gamma\%$ that a crack size at $T_\gamma(x_1)$ will be larger than x_1, see Fig. 2b. $T_\gamma(x_1)$ can be obtained from Eq. (8) by replacing $F_{T(x_1)}(\tau)$, and τ by $\gamma\%$ and $T_\gamma(x_1)$, respectively; with the result

$$T_\gamma(x_1) = \bar{t}(x_1)\exp[\sigma_z\Phi^{-1}(\gamma\%)] \tag{18}$$

in which σ_z is computed from Eq. (14) and the median service time $\bar{t}(x_1)$ to reach the x_1 has been obtained previously. $\Phi^{-1}(\ )$ is the inverse standardized normal distribution function. It is observed from Eq. (18) that the smaller the γ, the shorter the $T_\gamma(x_1)$.

The maintenance action date (calendar date) for the Ith aircraft, denoted by $D_\gamma(I)$, is given in the following integer format

$$D_\gamma(I) = D_n(I) + [T_\gamma(x_1) - t_n(I)](\bar{R})^{-1} \tag{19}$$

in which $D_n(I)$ is the recording date for the Ith aircraft with an accumulation of $t_n(I)$ FH. In Eq. (19), $[T_\gamma(x_1) - t_n(I)]$ is the number of flight hours remaining after $D_n(I)$, and $\bar{R}$, Eq. (17), is used to convert the remaining flight hours into the remaining number of calendar days (integer) to maintenance action.

Equation (19) can be used repeatedly for each fleet of aircraft flying similar load spectra. Then, the results for different fleets of aircraft can be combined to establish a required maintenance schedule for all aircraft. The maintenance date $D_\gamma(I)$ for the Ith aircraft serial number, computed from Eq. (19), is associated with a risk of $\gamma\%$. In other words, when the maintenance is performed on the Ith aircraft on the date $D_\gamma(I)$, there is a $\gamma\%$ probability that the crack size has exceeded the maintenance action flaw size x_1.

As observed from Eqs. (18) and (19), $D_\gamma(I)$ is shorter as the risk $\gamma\%$ becomes smaller. Equation (19) can be used to compute $D_\gamma(I)$ for $I = 1,2, \ldots, N$. Then these N data points are ranked in ascending order, denoted by $[D(\gamma, 1), D(\gamma, 2), \ldots, D(\gamma, N)]$, with $D(\gamma, j) \le D(\gamma, j+1)$. Such a data set can be used to determine the cumulative number of aircraft requiring the maintenance action as a function of the calendar date and the sequence for the maintenance action by aircraft serial number. The process can be repeated to obtain curves for different risk levels ($\gamma\%$).

In addition to $t_n(I)$, the usage severity parameters with respect to the fleet average load spectrum, such as the number of load exceedances over different levels, for the Ith aircraft can be used to estimate the crack growth curve from $\bar{a}_0$ up to $\hat{a}_n(I)$. This crack growth curve is schematically shown by a dashed-double dots curve in Fig. 2b. The second approach is to extrapolate this crack growth curve to x_1, denoted by $\bar{t}(I, x_1)$. This crack growth curve, i.e., from $\bar{a}_0$ to $\bar{t}(I, x_1)$, is the median crack growth curve for the Ith aircraft based on the individual aircraft tracking results. Then, the γ percentile service time $T_\gamma(I, x_1)$ for the maintenance action of the Ith aircraft as shown in Fig. 2b, is computed from Eq. (18) by replacing $T_\gamma(x_1)$ and $\bar{t}(x_1)$, respectively, by $T_\gamma(I, x_1)$ and $\bar{t}(I, x_1)$. Thus, the maintenance action time $T_\gamma(I, x_1)$ for the Ith aircraft can be updated periodically when more individual aircraft tracking results become available.

Demonstration and Evaluation

Reliability Centered Maintenance Analysis Method

The reliability centered maintenance analysis, τ_1, and τ_r, will be determined for a cutout in an aluminum-lithium check frame from an advanced fighter fuselage component. Three S are considered, including the effect of the stress gradient at the edge of the hole. The median crack growth curves for these three stress levels are computed from Eq. (5) using the ADAMSys[29] crack growth computer code, a durability analysis initial flaw size of $\bar{a}_0 = 0.005$ in. (0.127 mm), and a fighter load spectrum. A detectable flaw size limit of $a_D = 0.1$ in. (2.54 mm) (90% detection probability with 95% confidence) for the eddy current inspection technique is assumed. For the three stress levels, a_{cr} and the median service times to reach a_{cr} and a_D are given as follows: $S = 18.5$ ksi (127.5 MPa), $a_{cr} = 0.555$ in. (14.1 mm), $\bar{t}(a_{cr}) = 36{,}565$ FH, $\bar{t}(a_D) = 26{,}554$ FH; $S = 20.6$ ksi (142.0 MPa), $a_{cr} = 0.45$ in. (11.43 mm), $\bar{t}(a_{cr}) = 28{,}500$ FH, $\bar{t}(a_D) = 21{,}074$ FH; $S = 22.7$ ksi (156.5 MPa), $a_{cr} = 0.372$ in. (9.45 mm), $\bar{t}(a_{cr}) = 22{,}772$ FH, $\bar{t}(a_D) = 17{,}070$ FH.

The log standard deviation due to material crack resistance variability is estimated from laboratory crack growth data as $\sigma_{z1} = 0.131$. The log standard deviation due to usage severity of $\sigma_{z2} = 0.249$ is based on limited fleet tracking results and the method described above. The following σ_z values are used for the sensitivity studies: 1) $\sigma_z = \sigma_{z1} = 0.131$; 2) $\sigma_z = \sigma_{z2} = 0.249$; 3) $\sigma_z = [\sigma_{z1}^2 + \sigma_{z2}^2]^{1/2} = 0.281$; and 4) $\sigma_z = 0.337$. Historical data for a fighter aircraft are used to establish the maximum and minimum allowable risk limits[28]: $R_{a(\max)} = 5 \times 10^{-8}$/FH and $R_{a(\min)} = 10^{-8}$/FH. Other R_a values were also considered for sensitivity analyses. The conditional probability of a class A mishap is assumed to be $P_a = 0.9$, i.e., there is a 90% probability of losing the aircraft if the crack in the cutout grows to the critical crack size in flight. Other P_a values were also used for sensitivity analyses.

With the input data above, the reliability centered maintenance analysis procedures described in the previous section, the required τ_1, and τ_r, are presented in Figs. 3 and 4, respectively, as functions of R_a and σ_z. These results reflect a baseline stress level of 20.6 ksi (142.0 MPa), $x_1 = a_{cr} = 0.45$ in. (11.43 mm), and $P_a = 0.9$. It is observed from Figs. 3 and 4 that, as R_a decreases, the required τ_1 and τ_r become shorter. Also, as σ_z increases, the required τ_1 and the τ_r become shorter. The sensitivity of required τ_1 and τ_r, with respect to S, R_a, P_a, and σ_z, are presented in Figs. 5 and 6. In these figures, the baseline variable values are $S = 20.6$ ksi (142.0 MPa), $\sigma_z = 0.281$, $P_a = 0.9$, and $R_a = 5 \times 10^{-8}$/FH. However, the maximum value for variable P_a is 1.0. For instance, the required initial inspection intervals corresponding to 0.8 on the abscissa are based on $S = 0.8 \times 20.6 = 16.48$ ksi (113.6 MPa), $\sigma_z = 0.8 \times 0.281 = 0.225$, $P_a = 0.8 \times 0.9 = 0.72$, and $R_a = 0.8 \times (5 \times 10^{-8}) = 4 \times 10^{-8}$/FH, respectively.

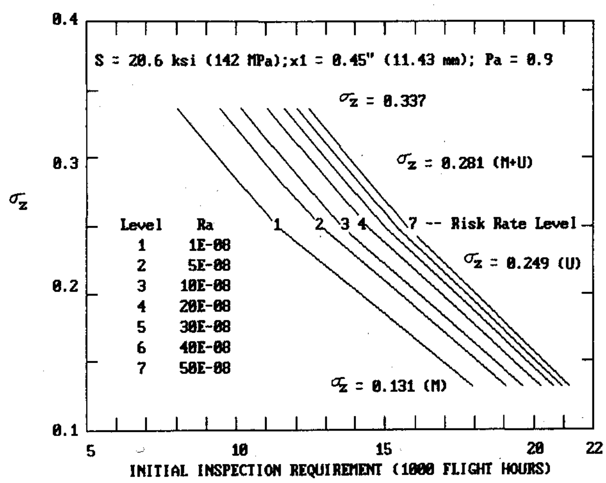

Fig. 3 Effect of σ_z and R_a on initial inspection interval.

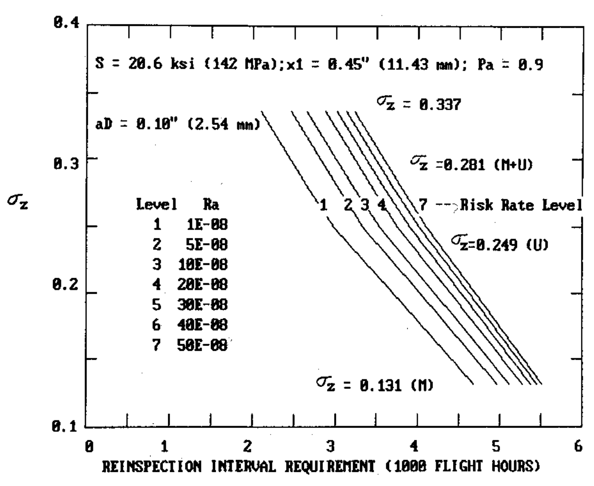

Fig. 4 Effect of σ_z and R_a on reinspection interval.

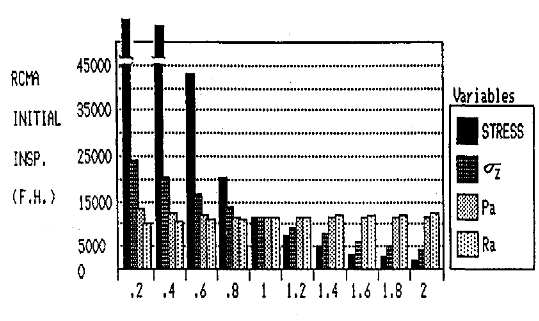

Fig. 5 Sensitivity of initial inspection interval to selected variables.

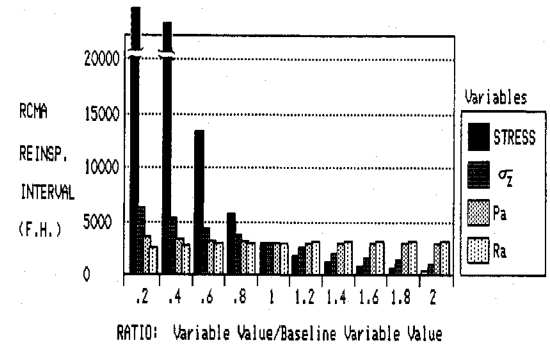

Fig. 6 Sensitivity of reinspection interval to selected variables.

It is observed from Figs. 5 and 6 that the required τ_1, and τ_r are influenced most by S, followed by σ_z. The effect of R_a and P_a on τ_1 and τ_r is less significant.

Fleet Maintenance Schedule

A fleet structural maintenance schedule will be developed for a 2-in.- (50.8-mm-) diam cutout in a fuselage bulkhead web for an advanced fighter aircraft. This cutout is a typical structural detail in the aluminum bulkhead [2024-T851; 5.5-in. (139.7-mm) plate, L-T], susceptible to fatigue cracking in service. The following parameters are used in the demonstration: 1) maintenance action flaw size $x_1 = 0.25$ in. (6.35 mm); 2) baseline $S = 27$ ksi (186.1 MPa); 3) $\pm 20\%$ stress variations, i.e., 21.6 ksi (148.9 MPa) and 32.4 ksi (223.4 MPa); and 4) risk levels $\gamma\% = 50$, 10, 1, and 0.1%.

Data for an individual aircraft tracking control point in the same general area as the 2-in.- (50.8-mm-) diam cutout are assumed to be applicable to the cutout for the demonstration. Individual aircraft tracking information for an aircraft fleet (e.g., 225 aircraft), as shown in Fig. 2a, are used to determine the crack growth life variabilities due to usage severity, σ_{z2}, and utilization rate, σ_{z3}; with the results $\sigma_{z2} = 0.23$ and $\sigma_{z3} = 0.20$. The crack growth life dispersion due to material crack growth resistance is assumed to be $\sigma_{z1} = 0.10$. Thus, σ_z is computed from Eq. (14) as 0.32. A fleet size of 225 aircraft is considered, i.e., $N = 225$, and $\bar{R}$ of 0.459 FH/day is obtained.

Based on the fleet average load spectrum, the median flight hours to reach the maintenance action flaw size $x_1 = 0.25$ in. (6.35 mm) and a_{cr} are given as follows for three different maximum stress levels: 1) $S = 21.6$ ksi (148.9 MPa), $a_{cr} = 0.812$ in. (20.62 mm), $\bar{t}(x_1) = 8390$ FH, $\bar{t}(a_{cr}) = 10{,}915$ FH; 2) $S = 27$ ksi (186.1 MPa), $a_{cr} = 0.52$ in. (13.2 mm), $\bar{t}(x_1) = 4800$ FH, $\bar{t}(a_{cr}) = 5700$ FH; and 3) $S = 32.4$ ksi (223.4 MPa), $a_{cr} = 0.361$ in. (9.17 mm), $\bar{t}(x_1) = 3045$ FH, $\bar{t}(a_{cr}) = 3330$ FH. A durability $\bar{a}_0 = 0.005$ in. (0.127 mm) (corner flaw) is used. Equation (18) is used to compute the time, $T_\gamma(x_1)$, to reach the maintenance action flaw size $x_1 = 0.25$ in. (6.35 mm) for different risk levels $\gamma\%$. The results for the baseline $S = 27$ ksi (186.1 MPa) are as follows: $\gamma = 0.1$, $T_\gamma(x_1) = 2169$ FH; $\gamma = 1$, $T_\gamma(x_1) = 2639$ FH; $\gamma = 10$, $T_\gamma(x_1) = 3452$ FH; $\gamma = 50$, $T_\gamma(x_1) = 4800$ FH.

Based on the results presented above and the procedures described in the previous section, the calendar dates, $D_\gamma(I)$, $I = 1, 2, \ldots, N$, for structural maintenance of each aircraft in the fleet for selected γ, can be computed from Eq. (19). These dates (and corresponding aircraft serial numbers) are ranked in ascending order, and the cumulative number of aircraft in a fleet requiring structural maintenance actions is plotted against the calendar year. The results for a baseline stress level of 27 ksi (186.1 MPa), $\sigma_z = 0.32$ and $\bar{R} = 0.459$ FH/day are presented in Fig. 7. In Fig. 7 the fleet maintenance schedule is shown for four risk levels (i.e., $\gamma\% = 0.1$, 1, 10, and 50%). Each curve describes, for a selected risk level, the cumulative number of aircraft requiring structural maintenance on a given calendar date. The risk level herein refers to the probability that the flaw size will exceed the mainte-

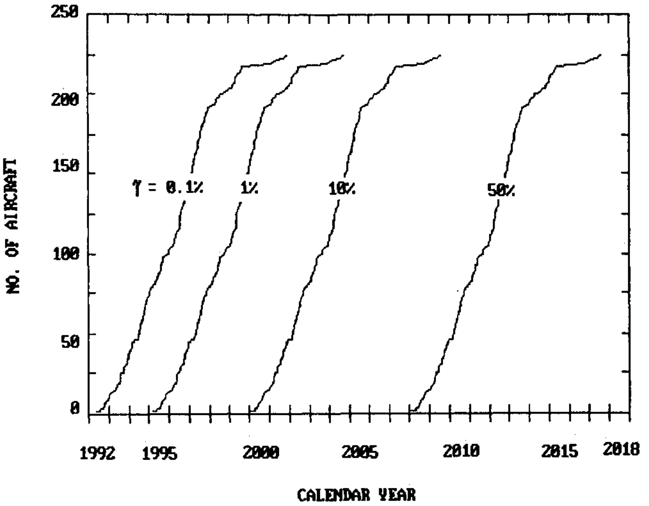

Fig. 7 Effect of risk level on fleet maintenance schedule for a structural detail.

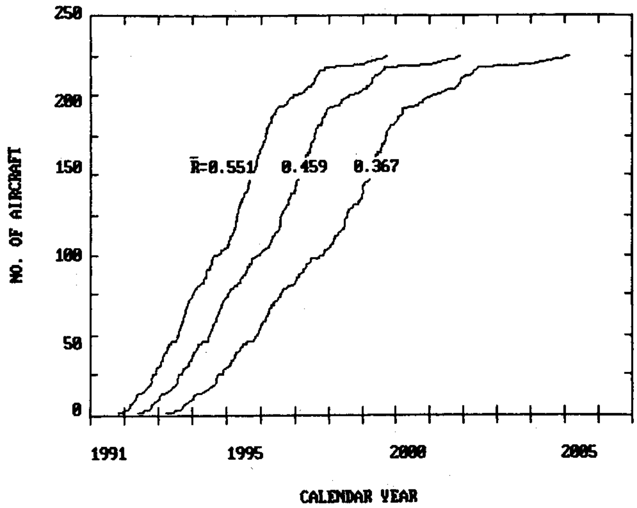

Fig. 9 Effect of utilization rate variation on fleet maintenance schedule for a structural detail.

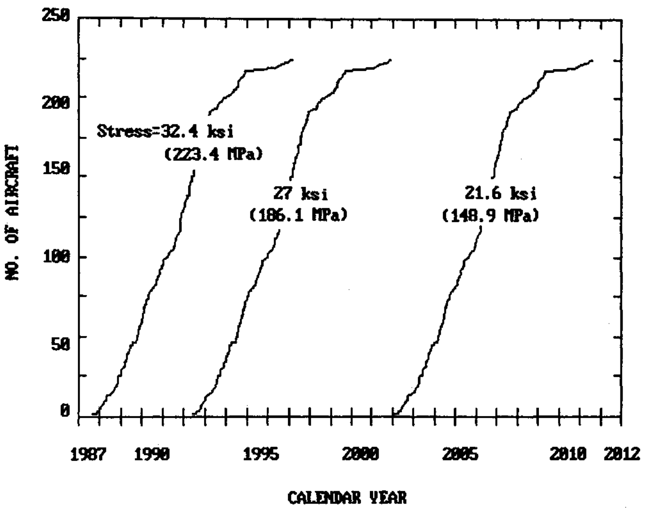

Fig. 8 Effect of stress level variations on fleet maintenance schedule for a structural detail.

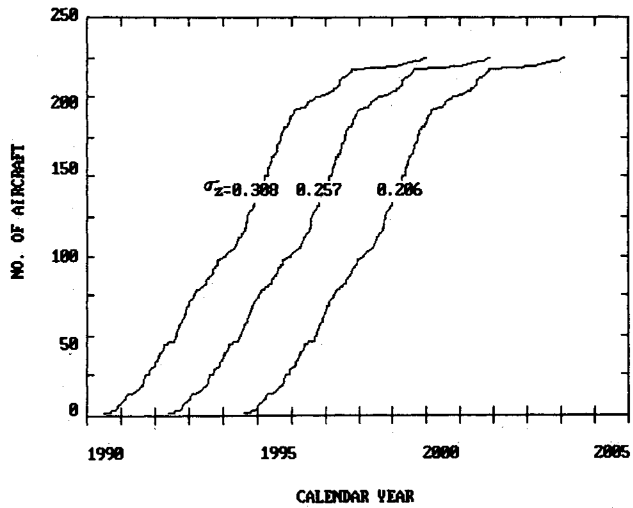

Fig. 10 Effect of crack growth life variability on fleet maintenance schedule for a structural detail.

nance action flaw size x_1 = 0.25 in. (6.35 mm) on the maintenance action calendar date. The results presented in Fig. 7 can also be used to evaluate fleet maintenance tradeoffs. For instance, if the allowable risk level is increased from 1 to 10%, structural maintenance can be delayed as shown by comparing the 1 and the 10% risk curves in Fig. 7.

The structural maintenance requirements are plotted in Fig. 8 for three different stress levels, i.e., the baseline stress level of 27 ksi (186.1 MPa) and ±20% variations. Figure 8 reflects $\bar{R}$ = 0.459 FH/day, x_1 = 0.25 in. (6.35 mm), σ_z = 0.32, and $\gamma\%$ = 0.1% risk level. It is observed from Fig. 8 that with the nominal stress level of 27 ksi (186.1 MPa), structural maintenance would not be required until 1992. However, if the stress level is increased to 32.4 ksi (223.4 MPa) (20% increase), fleet structural maintenance would be required in 1987.

For the baseline stress level of 27 ksi (186.1 MPa), x_1 = 0.25 in. (6.35 mm) and 0.1% risk level, the effect of ±20% variations in the nominal utilization rate on structural maintenance requirements is shown in Fig. 9. As expected, the higher the utilization rate, the sooner the maintenance action is required. Figure 10 presents the fleet maintenance schedules for the effect of ±20% variations in σ_z. In Fig. 10, the following parameters were used: nominal σ_z = 0.32, risk level = 0.1%, stress level = 27 ksi (186.1 MPa), and $\bar{R}$ = 0.459. As observed from Fig. 10, the larger the crack growth life dispersion is, the earlier maintenance action is required.

The results shown in Figs. 7–10 can be used to estimate the number of aircraft in the fleet requiring maintenance action during a given calendar period (e.g., 1992–1994, 1993, and the fourth quarter of 1994) for a given risk level. These results can also be tabulated for selected calendar periods to provide specific dates for maintenance action by individual aircraft serial numbers. This information is useful for planning aircraft utilization, planning maintenance, and forecasting resource requirements to accomplish the maintenance actions. In the demonstration above, the effect of S dominates, followed by $\gamma\%$, $\bar{R}$, and σ_z. Finally, the fleet maintenance plan described above should be updated periodically as more individual aircraft tracking results become available.

Conclusions

A stochastic crack growth analysis, based on the lognormal random variable model, has been described and demonstrated for practical applications to aircraft structural maintenance. The crack growth life dispersion due to material, usage severity, and aircraft utilization rate has been accounted for. The methodology for determining the crack growth life dispersion due to various sources using available IAT results has been presented.

The reliability centered maintenance analysis method has been described, demonstrated, and evaluated using a cutout in a metallic cheek frame. The sensitivity of inspection intervals with respect to S, σ_z, R_a, and P_a, has been investigated.

For the same percentage of variation in the baseline variable values, it was shown that S, followed by σ_z, affects the results more than either R_a or P_a.

A method of scheduling aircraft fleet maintenance on a calendar year basis in terms of risk has been presented. The effect and sensitivity of various factors on the structural maintenance schedule were investigated. It was found that the stress level has the biggest effect on the structural maintenance schedule, followed by the risk level. The effect of the crack growth life dispersion and the utilization rate on the fleet maintenance schedule is also significant.

This article emphasizes the maintenance schedule for the initial inspection and practical applications. In determining the reinspection interval, a_D for a particular nondestructive evaluation (NDE) system is used to be consistent with current practice. a_D is a crack size that can be detected by a particular NDE system with 90% probability and 95% confidence. Strictly speaking, instead of using a_D, the probability of crack detection (POD) curve should be used,[1,8] although it has not been adopted in practice. As the fleet of aircraft ages and service life extension is needed, the POD curve becomes more important, and other methods[1,4,5] should be used. Finally, the fleet maintenance schedule should be updated periodically as more individual aircraft tracking results become available.

References

[1]Lincoln, J. W., "Risk Assessment of an Aging Military Aircraft," *Journal of Aircraft*, Vol. 22, No. 8, 1985, pp. 687–691.

[2]Smith, H., Saff, C. R., and Christian, T. F., "Structural Risk Assessment and Aircraft Fleet Maintenance," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 31st Structures, Structural Dynamics and Materials Conference* (Long Beach, CA), AIAA, Washington, DC, 1990, pp. 263–274.

[3]Yang, J. N., "Statistical Estimation of Service Cracks and Maintenance Cost for Aircraft Structures," *Journal of Aircraft*, Vol. 13, No. 12, 1976, pp. 929–937.

[4]Berens, A. P., Hovey, P. W., and Skinn, D. A., "Risk Analysis for Aging Aircraft Fleets, Volume I—Analysis," Vol. I, Wright Lab., Wright-Patterson AFB, WL-TR-91-3066, Dayton, OH, 1991.

[5]Yang, J. N., and Trapp, W. J., "Reliability Analysis of Aircraft Structures Under Random Loading and Periodic Inspection," *AIAA Journal*, Vol. 12, No. 12, 1974, pp. 1623–1630.

[6]Yang, J. N., and Chen, S., "Fatigue Reliability of Gas Turbine Engine Components Under Scheduled Inspection Maintenance," *Journal of Aircraft*, Vol. 22, No. 5, 1985, pp. 415–422.

[7]Yang, J. N., and Chen, S., "An Exploratory Study of Retirement-for-Cause for Gas Turbine Engine Components," *Journal of Propulsion and Power*, Vol. 2, No. 1, 1986, pp. 38–49.

[8]Yang, J. N., and Chen, S., "Fatigue Reliablity of Structural Components Under Scheduled Inspection and Repair Maintenance," *Probabilistic Methods in Mechanics of Solids and Structures*, edited by S. Eggwertz, Springer-Verlag, Berlin, 1985, pp. 559–568.

[9]Yang, J. N., and Donath, R. C., "Statistical Fatigue Crack Propagation in Fastener Holes Under Spectrum Loading," *Journal of Aircraft*, Vol. 20, No. 12, 1983, pp. 1028–1032.

[10]Provan, J. W. (ed.), *Probabilistic Fracture Mechanics and Reliability*, Martinus Nijhoff, Dordrecht, The Netherlands, 1987.

[11]Yang, J. N., Hsi, W. H., and Manning, S. D., "Stochastic Crack Growth Models for Application to Aircraft Structures," *Probabilistic Fracture Mechanics and Reliability*, edited by J. W. Provan, Martinus Nijhoff, Dordrecht, The Netherlands, 1987, pp. 171–212.

[12]Palmberg, B., Blom, A. F., and Eggwertz, S., "Probabilistic Damage Tolerance Analysis of Aircraft Structures," *Probabilistic Fracture Mechanics and Reliability*, edited by J. W. Provan, Martinus Nijhoff, Dordrecht, The Netherlands, 1987, pp. 47–128.

[13]Provan, J. W., "Probabilistic Approaches to the Material-Related Reliability of Fracture-Sensitive Structures," *Probabilistic Fracture Mechanics and Reliability*, edited by J. W. Provan, Martinus Nijhoff, Dordrecht, The Netherlands, 1987, pp. 1–44.

[14]Yang, J. N., Salivar, G. C., and Annis, C. G., "Statistical Modeling of Fatigue Crack Growth in a Nickel-Based Superalloy," *Journal of Engineering Fracture Mechanics*, Vol. 18, No. 2, 1983, pp. 257–270.

[15]Salivar, G. C., Yang, J. N., and Schwartz, B. J., "A Statistical Model for the Prediction of Fatigue Crack Propagation Under a Block Type Spectrum Loading," *Journal of Engineering Fracture Mechanics*, Vol. 31, No. 3, 1988, pp. 371–380.

[16]Lin, Y. K., and Yang, J. N., "On Statistical Moments of Fatigue Crack Propagation," *Journal of Engineering Fracture Mechanics*, Vol. 18, No. 2, 1983, pp. 243–262.

[17]Lin, Y. K., and Yang, J. N., "A Stochastic Theory of Fatigue Crack Propagation," *AIAA Journal*, Vol. 23, No. 1, 1985, pp. 117–124.

[18]Lin, Y. K., Wu, W. F., and Yang, J. N., "Stochastic Modeling of Fatigue Crack Propagation," *Probabilistic Methods in Mechanics of Solids and Structures*, edited by S. Eggwertz and N. C. Lind, Springer-Verlag, Berlin, Jan. 1985, pp. 103–110.

[19]Virkler, D. A., Hillberry, B. M., and Goel, P. K., "The Statistical Nature of Fatigue Crack Propagation," *Journal of Engineering Materials and Technology*, American Society of Mechanical Engineers, Vol. 101, No. 2, 1979, pp. 148–152.

[20]Bogdanoff, J. L., and Kozin, F., *Probabilistic Models of Cumulative Damage*, Wiley, New York, 1985.

[21]Bogdanoff, J. L., and Kozin, F., "Probabilistic Models of Fatigue Crack Growth—II," *Journal of Engineering Fracture Mechanics*, Vol. 20, No. 2, 1984, pp. 255–270.

[22]Ortiz, K., "On the Stochastic Modeling of Fatigue Crack Growth," Ph.D. Dissertation, Stanford Univ., Stanford, CA, 1985.

[23]Spencer, B. F., Jr., and Tang, J., "A Markov Process Model for Fatigue Crack Growth," *Journal of Engineering Mechanics Division, ASCE*, Vol. 114, 1988, pp. 2134–2157.

[24]Yang, J. N., and Manning, S. D., "Stochastic Crack Growth Analysis Methodologies for Metallic Structures," *Journal of Engineering Fracture Mechanics*, Vol. 37, No. 5, 1990, pp. 1105–1124.

[25]Yang, J. N., and Manning, S. D., "Demonstration of Probabilistic-Based Durability Analysis Method for Metallic Airframes," *Journal of Aircraft*, Vol. 27, No. 2, 1990, pp. 169–175.

[26]Manning, S. D., and Yang, J. N., "USAF Durability Design Handbook: Guidelines for Analysis and Design of Durable Aircraft Structures," Air Force Wright Aeronautical Lab., TR-88-3119, Wright Patterson AFB, Dayton, OH, Feb. 1989.

[27]Miller, M. S., and Gallagher, J. P., "An Analysis of Several Fatigue Crack Growth Rate (FCGR) Description," *Fatigue Crack Growth Measurement and Data Analysis*, American Society for Testing and Materials, ASTM-STP-738, 1981, pp. 205–251.

[28]Manning, S. D., Yang, J. N., Pretzer, F. L., and Marler, J. E., "Reliability Centered Maintenance for Metallic Airframes Based on a Stochastic Crack Growth Approach," *Advanced in Fatigue Lifetime Predictive Techniques*, edited by M. R. Mitchell, American Society for Testing and Materials, ASTM-STP 1122, 1992, pp. 422–434.

[29]Roach, G. R., McComb, T. H., and Chung, J. H., *ADAMSys User's Manual*, Structures and Design Dept., General Dynamics, Fort Worth Div., Fort Worth, TX, July 1987.